

Philosophy 324A

Philosophy of Logic

2016

Note Fifteen

CATCHING OUR BREATH ABOUT PLURALISM (AND CLEARING UP SOME OF THE MURK OF YESTERDAY'S IDEAS)

Try to keep in mind that historically it is motivated by a desire to take some of the sting out of the multiplicity and strife problem. It is intended as a kind of conflict-resolution device to being some measure of reconciliation to the tensions that afflict modern logic. It is an oddly structured background.

- Never has logic been done so well as now.
- Never have we had so much of it, and of such wide range.
- On the face of it at least, never have there been such high levels of conflict among logicians.
- Never in logic's long history has there been so little notice of this multiplicity and so little effort to remove or ameliorate the strife it appears to breed.¹

One of the big questions posed by the multiplicity and strife problem is whether it constitutes a "moral hazard" for realism. This has produced various answers and – guess what! – the answers conflict. Here are five of the more commonly given ones

- *Nihilism or irrealism*: There are no logical facts, not even the ones that logicians make up for their systems.
- *System-relativity realism*: The only logical facts are the facts relative to some or other system of logic. These are genuine facts but only in the system that sanctions them. Accordingly, the correct form of a truth claim in logic is that A is true-in-system L.
- *Ambiguity realism*: There are as many different facts about matters fashioned under the concept K as there are different meanings of the expression "K". Each logical fact is a real-world fact concerning the matters that fall under the concept specified by the particular meaning of the term that expresses the concept.
- *Species realism*: The real facts about matters falling under some concept K hold true for one (or more) species of the concept K. If there is a genus of which these instantiations of K, it *needn't* be the case that any fact of logic pertains to it. (For example, if the generic

¹ There are exceptions, of course. One of the more recent is Steward Shapiro's *Varieties of Logic*, New York: Oxford University Press, 2014.

concept itself is too vague or imprecise or polysemous, it might not qualify as a safe site for any solid fact of logic.)

- *Hopeful realism*: System-facts are made up in the hopes that they might eventually be load-bearing in a theory that has good realist credentials. (For example, Riemann thought that no theorem of his geometry was actually true of real space. It took relativity theory to show that they actually were).

We might note before moving on to B&R pluralism that each of these five answers is pairwise inequivalent to the others and that the nihilism answer conflicts with the other four. Shades again of multiplicity and strife!

B&R PLURALISM

How might we summarize B&R pluralism, as we have it so far from the first seven chapters of *Logical Pluralism*? It would go something like this:

- (1) *The centrality of consequence thesis*: The concept that lies at the heart of logic is the concept of logical consequence (or of following, of necessity, from; or logical implication; or entailment).
 - (1a) *Possible corollary thereof?* Any question about logic's other key properties can be answered (in principle) by careful examination of the consequence relation.
- (2) *The ambiguity thesis*: The expression "logical consequence" is ambiguous. There are specifiably different senses that it possesses.
- (3) *The multiplicity of consequence thesis*: For each different meaning of "logical consequence", there is a different concept of logical consequence.
- (4) *The species thesis*: Corresponding to each different *concept* of logical consequence there is a different *species* of the consequence relation.

SUBJECT-MATTER PLURALISM

In earlier postings and class discussions we have raised doubts about each of the B&R theses. Doubts are doubts, not conclusive refutations. So we shouldn't think that these matters are now closed. Part of your job is to keep them in mind and to reflect on them further. Part of my job is to keep things moving. So let's get a move on, but not before a last brief word about the centrality question along the lines laid out in note #14. There are two different and inequivalent questions to ask about logic:

- Which concepts are at the heart of logic?
- With what subject-matters is logic concerned?

Perhaps the first thing to notice about the concept of pluralism is that it itself has a character that positively invites *self-instantiation*. To get the ball rolling, let's mark the difference between

- *Subject-matter pluralism* (SMP)

and

- *Heart-of-logic pluralism* (HLP)

One of the key mistakes in logical theory is thinking that either kind of pluralism directs traffic in the other as a matter of course. Sometimes they do, sometimes not. For an example of when SMP doesn't effect HLP, let's go back to Aristotle. Consider a syllogism in *Barbara*

- (1) All Greeks are humans
- (2) All humans are mortal
- (3) All Greeks are mortal [QED]

Bearing in mind that a syllogism is a valid argument meeting the further conditions on syllogisms, for example, that they contain no redundant premisses, consider the following argument.

- (a) All ocelots are four-legged
- (b) All Greeks are human
- (c) All humans are mortal
- (d) All Greeks are mortal [QED]

Clearly this second argument is valid. Just as clearly, it is not a syllogism. The fact that it is not lays no glove whatever on the nature of the consequence relation that underlies it.

The subject-matter of Aristotle is in one way or another about *sylogisticity*. Validity lies at the heart of syllogistic, but not one fact about syllogisticity changes any prior fact about validity. Syllogistic logic is about lots of different things – syllogism as-such, syllogisms in-use, refutations, scientific demonstrations, redundancy, relevance, etc. – but nothing it says about these things requires that we acknowledge two or more concepts of logical consequence (or validity.)

For an example of when SMP does imply HLP, let's go back to Lewis' S1-S5. We can easily agree that each of them treats logical necessity and possibility differently in each of these systems. Since necessity and possibility are *defining conditions* on strict implication (which is Lewis' name for logical consequence), we can hardly doubt that at the heart of each of these five different modal systems lies a *different* concept of logical consequence.

Since it is pretty well accepted that S4 and S5 capture different but intuitively plausible notions of logical necessity and possibility, it is easy to see a corresponding difference between S4-strict implication and S5-strict implication. On the other hand, since there is lots of uncertainty about whether S1-S3 capture different but recognizably *bona fide* concepts of necessity and possibility, this same uncertainty spreads to the S1, S2, and S3 conceptions of logical consequence.

Note well that the easy route to logical consequence pluralism (HLP) is via the subject-matter pluralism (SMP) of the conditions that define it. In which case, SMP wears the trousers

for HLP in all contexts in which SMP applies to *defining conditions* on the consequence relations.

- *A closing speculation*: Is this why modal logics don't make the scene in *Logical Pluralism*. To preserve the primacy of HLP for logical consequence?

PARADIGM-RECOGNIZABILITY PLURALISM

Let's abbreviate this to PRP. In some ways, our old friend the CC-line sets up an architecture for this third kind of pluralism. Let's start with the intuitive concept of following, of necessity, from. Everyone who speaks English and has had some contact in school with geometry has a good working knowledge of how this concept operates in English. But many fewer of this same very large number would be able to snap out a *definition* of following, of necessity, from. Their knowledge of its meaning is for the most part implicit.

Suppose we asked them to consider the following definition: B follows, of necessity from A if and only if it is not logically possible for A to be true and B not. How likely is it that the definition wouldn't be readily accepted? You're right. Not likely at all. Consider the definition the *analysis* of the intuitive concept it defines. So far, so good. The only difference in meaning between the two is that the meaning of the intuitive concept is implicit, and it – the very same meaning – is explicit in the analyzed concept. The analyzed concept nicely passes the concept-recognition test (CRT): the intuitive concept is recognizable in the analyzed one; the relation of following, of necessity, from is recognizably present in the analysis which says that it is not logically possible for the proposition that follows to be false when the proposition from which it follows is true.

Right away, this gives us pluralistic pause. Is the present analysis of the intuitive concept the only plausible one? Suppose we tried something different, along the following lines:

- B follows, of necessity, from A if and only if every case whatever in which A is true, B is too.

Take note of the difference: The first is an expressly modal analysis, whereas the second one is expressly quantified and not modal one. Do we now have two equi-plausible analyses of the intuitive concept? If so, does this give us analytic-pluralism at the point of analysis? If so, the analyzed concept gives different senses of logical consequence.

Some philosophers see the intuitive concept as paradigmatic, and the analyzed concept as derivative. Others see the analyzed concept as the paradigmatic one. For those who do, there is a good recognizability question now to ask of them.

- Is there a clarifying improvement of the paradigm-concept(s) to be got from a CC-*explication* of it (them)?

Here, too, answers differ, but among logicians there is a positive tilt towards explication. Taking Quine as an example, we could plump for the explication afforded by classical logic, ramifying in turn to classical entailment and classical deducibility. In which case the concept of following, of necessity, from would “pluralize” to semantic and syntactic notions of following, of necessity from.

When we consider all the alternatives to classical consequence – intuitionist consequence, linear consequence, labelled deduction consequence, etc. – explicated consequence ramifies further and further enlarges pluralism’s embrace.

With these ramifications further questions also come.

- Is there in these various explication of the paradigm analytic concept of following, of necessity, from the recognizable presence of the paradigm-concept?

If so, pluralism’s corresponding enlargement is home and dry.

Let’s go now to the next step, using the example of Priest. Suppose we thought that some select few contradictions really were true. Then we would seek for the paradigm-concept(s), whether the intuitive one, or the analyzed or explicated ones, a radical make-over – a rational reconstruction – in which negation wouldn’t in all cases be a truth-value flipper. Why? Because, for just these cases, A could take the truth value true and the negation operator *wouldn’t* make $\sim A$ false. Wow!

There are two basic reactions to this. One is Quine’s:

- Priest’s logic makes the concept of negation unrecognizable, and in so doing does the same for the concept of following, of necessity, from.

The other Is Priest’s own:

- On the contrary, the dialethic reconstruction of paradigm-negation is a signal advance in the logic of paradoxical consequence, and an overall liberation of logic from the bondage of ossified paradigms.

We see in this PRP approach a clear link between left-to-right movement along the CC-line is an unmistakable and an expansion of a concept’s pluralistic reach. And notice, as well, the ease with which this pluralism arose without having to go to the example set by modal logic.

It is now time to take the measure of chapter eight. That is your task for next Tuesday. Your Thursday task is to attend Ori Simchen’s generously offered guest lecture. When I return from Korea, the Tuesday that follows will be reserved for chapter 9. Happy sails!